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The Texas Mathematics Teachers' Bulletin

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The benefits of education and of useful knowledge, generally diffused through a community, are essential to the preservation of a free government.

Sam Houston

Cultivated mind is the guardian genius of democracy. . . . It is the only dictator that freemen acknowledge and the only security that freemen desire.

Mirabeau B. Lamar

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This Bulletin is open to the teachers of mathematics in Texas for the expression of their views. The editors assume no responsibility for statements of facts or opinions in articles not written by them.

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MATHEMATICS FACULTY OF THE UNIVERSITY OF TEXAS

VITALIZING THE TEACHING OF ALGEBRA AND GEOMETRY

By S. M. Sewell, Southwest Texas Normal

The subject of this paper is one that should closely engage our attention. Vitalization is the key word; without it our true work is but half accomplished. Putting life into the work means arousing interest, and without proper interest we can accomplish little. A student's progress in any subject is, in general, commensurate with his interest in that work.

In mathematics, as in other school subjects, much has been done in the last few years toward making the student feel a lively interest in the work. The work in this department, as we all know, is generally considered dry and formal; it should not be so. If we put forth sufficient effort, there is no reason why this work should not be made just as much alive as agriculture, physics, domestic science, or manual training.

I am expected to deal with this subject as it relates to algebra and geometry. Most that I shall have to say, however, will be more specially applicable to geometry, for two reasons: first, that students, as a rule, find more difficulty in entering into the merits and pleasures of geometry than of algebra, and second, that teachers, as a rule, place more emphasis upon algebra and find it easier to teach.

Considering the limited time at our disposal for the discussion of this subject, it seems that our best plan should be simply to set forth a few of the principal means of injecting more life into the work of teaching mathematics. Let us treat the subject under the following four heads: 1. Motivation, 2. History, 3. Applications, and 4. Classroom tactics.

1. *Motivation.* This is an age of materialism. One constantly stops to say "wherein will it profit me?" We hesitate to engage in a thing merely for curiosity or pastime. Even the school boy is learning to stop and say, "Shall I

ever need this?" When we have a direct motive in all our activities we enter into them with life and vigor. Early in the course in algebra and geometry we should take ample time to set forth good and sufficient reasons for the pursuance of the subject. There was a time when we pursued our school work primarily through the pure love of knowledge. Not so much stress was then placed upon credits and diplomas. But now the student is much more enthusiastic in his work if he can give you a reason for his studying, other than to get credit on the course. We should point out to the student that algebra is an indispensable tool for further mathematical work; that geometry is the foundation of all architectural and decorative design and that these subjects afford the best possible means for developing ideas of form, accuracy, logical sequence, and self mastery.

D. E. Smith says, "Foremost as a reason for studying geometry has always stood, and will always stand, the pleasure and the mental uplift that comes from the contact with the great body of human learning, and particularly with the exact truth that it contains." Many things we learn for the pleasure of knowing. This is largely true with music and art, for which people make greatest sacrifice. Furthermore, one cannot become educated in the true sense by pursuing one line of thought alone. Such phases of mental culture should be presented to the student, and he should be induced, if possible, to have a real motive in his work other than to secure scholastic credit.

2. *History.* Only in the last few years have we seemed to realize that a little historical sketch thrown in here and there adds much to the life and interest of the work. A student will study the forty seventh problem of Euclid with keener interest if he knows that it played an important part in the building of the Pyramids of Egypt or of King Solomon's Temple. The student's eyes will dance with delight when you reveal the fact that there was a time when geometry was one of four subjects which constituted the school curriculum, or that the value of π has been

calculated to 707 decimals, or that a single algebraic solution has been developed that would cover 200 pages. To weave in these little historical items consumes almost no time, and often accomplishes much toward fixing in mind certain fundamental facts. The student should know something of when and by whom the development of the subject has flourished. Few students can fail to feel a new thrill of interest when the historical sidelights are occasionally thrown upon our work. Library references or current magazine articles may well be given. Occasionally it is good to read a short sketch to the class with appropriate comment.

Pictures of mathematicians should adorn the walls of the classroom. I have found that such pictures are hard to get, but we ought to have them. No one doubts for a moment the vitalizing effect upon a literature class of a life-size picture of Longfellow or Shakespeare; or upon a history class of the picture of Washington or Napoleon; or upon the Latin class of a statue of Caesar or Constantine. Then why should we not inspire our mathematics students with the pictures and statues of such men as Newton, Euclid, Archimedes, and Pascal?

4. *Applications.* The tactful teacher can do much toward vitalizing his work by stressing in various ways the practical applications of the subject. This is more generally true of geometry than of algebra. I think, however, that we sometimes go to the extreme in trying to show the student the practical benefit in everything he is doing. The student is in a dangerous condition when he has reached the point in his scholastic career where he is constantly stopping to ask if *this* will be of any practical benefit to him. The average student cannot see in advance, neither can his teacher point out, wherein he will have need of everything he is expected to learn. Again, we encounter many things that are not directly practical, but are worth a great deal indirectly. The student should be led to feel that his accomplishments are well worth his time and effort, regardless of practical applications. *Culture* is worth consideration. One might as well say he need not read poetry unless

he intends to become a poet as to say that he need not study algebra or geometry unless he expects to become an engineer. Someone has said, "Without mathematics no one can fathom the depths of philosophy. Without philosophy no one can fathom the depths of mathematics. Without these two no one can fathom the depths of anything."

We should beware of using "far-fetched" applications. Students readily see the folly of it and are disgusted at the effort. Yet nothing will contribute more toward a good lively interest in the work than to use a few simple, attractive, and commonplace applications and illustrations of principles that we encounter here and there. The resourceful teacher will find it much better to put forth such applications and illustrations out of the immediate environment than to take them bodily out of the text-book, though the exercises be intrinsically no better. For example, let the teacher take the class out of doors and with a measuring tape measure some inaccessible line by use of congruence of triangles. On discussing the diagonals of a rectangle, make a miniature yard gate and illustrate the use of its diagonal brace. Have the pupils construct some decorative geometric design. Go out and measure the height of a tree or a building by the length of its shadow by use of similarity of triangles. Show how the similarity of polygons may be used in enlarging a map by means of a little rubber band. (Illustration.) Send the students to the physics laboratory or the manual training room to find instances of application. It is easy to find plenty of this kind of work in geometry—in fact, we should be careful not to do so much of this as to neglect the proper development of form, exactness of expression, and logical sequence.

4. *Classroom Tactics.* One's success in teaching mathematics depends as much upon his knowledge of how to present the subject-matter as upon his knowledge of the subject-matter itself. Geometry is hard to teach. More than almost any other subject does it require special tact and forethought. It takes a student at least three weeks to get his bearings. If by that time he has not acquired the proper attitude toward the subject and the right appreciation of

what he is doing, he would about as well lay it aside for the time and afterwards start it again. It is the province of the teacher to lead the student into a fondness for his work and a proper appreciation of what is expected of him. For a teacher to say that this is an easy task is but an open confession of his failure. Just here individual tact plays a great part. However, we should know well what things to emphasize most. On this point we wish to recommend the classification of propositions as given in the Geometry Syllabus by the National Committee of Fifteen. We sometimes make the error of thinking that we must cover everything given in the text. We must dwell upon certain things, even if we have to leave certain other things untouched. We should also know the proper proportion of oral and written work. A student may learn to give a creditable demonstration orally, and yet cannot write it; and *vice versa.* Form, language, manner of attack, responsiveness, power of expression, and self-mastery are some of the things that we must keep constantly in view.

Our class work should be so executed as to keep as nearly as possible all the class thinking during the entire class period. It's a mistake to give much individual assistance on particular problems when the majority of the class are not otherwise intensively engaged. Only a few are interested in what you are saying, and to most of the class the time is wasted. Just here we wish to emphasize the value of simple, snappy, interesting drill exercises. Let the assignment of lessons include definite problem work, and discuss in class the general principles involved in these problems; then give lively drill exercises involving these principles. If then there are a few students who cannot apply the principles to certain problems, let them dig it out alone or let it go, unless you are so situated as to render a little assistance outside of class. The student in mathematics should be able to do a few things *well* and in *short time.* The Courtis Standard Tests are doing much toward bringing about a higher proficiency in the fundamentals of arithmetic. Why should we not have such standardization tests in algebra and geometry? Especially in algebra we may make great use of the drill upon the simpler and more useful operations. If more time were spent in this way, and less upon problems in simultaneous equations, evolution, etc., our students would be better prepared for further mathematical work. Even in geometry one may employ the drill to good advantage. When well along in the course we have tried the drill in the form of a contest, when more than one hundred propositions would be accurately recited in twenty minutes.

If we will tear loose from some of the customs of our grandfathers, and throw more individuality into our work, in the light of modern advancement, there is no reason why algebra or geometry should not be made as vital as any other subject in the high-school curriculum.

MATHEMATICAL BOOKS IN THE SCHOOL LIBRARY

Many schools possess no library and perhaps cannot afford even a modest one, but the modern tendency toward special methodology, toward the development of aids, props, appurtenances, even at the expense, at times, of thorough grounding, and of the proper emphasis upon personality and scholarship—has resulted in the fairly widespread dissemination of the feeling that no modern American school is complete without a library. The character and use of the library is another matter.

An efficient publishing house with alert agents looks upon the opening of a new school as an opportunity for salesmanship, the chance to dispose of many sets and series, quite beyond the scope or ambition of the several teachers individually. An edition of some encyclopedia, an atlas, a dictionary, two or three extensive sets on history from the earliest times, and some more on American and British authors, or perhaps, literature in general, form a natural nucleus for a school library of reference books. To be sure, these books when bought, and eventually paid for, may never be looked at—in fact the pupils may never have the opportunity even to learn what the books claim to deal with, but they look well on the shelves of the office, and give a literary and businesslike air to the institution.

The library may be of another sort entirely. It may comprise a large number of books of fiction and a few of travel, adventure or of practical mechanics, such as might appeal to the fancy of growing boys and girls. Even so it may be little patronized, or the relatively large number of books taken out may indicate a response to the artificial stimulation of home reading through the granting of extra credits in history, English, or some other course for books read out of class when selected from certain approved lists. For the average pupil there is likely to be too much of compulsion and drudgery in the mental association of the school building and school books for him or her to seek for enjoyment in any book loaned from a school library.

The child may be unusual to the extent of thoroughly enjoying to read, with an omnivorous literary appetite and even be a regular frequenter of a local public library and yet the school library may serve only as a last resort.

A library may include books for the teacher, books not to be regarded as reference works, consulted on special items of dispute, but rather consecutive treatises dealing either with the subject-matter being taught or with the methods of teaching. These are of course more or less authoritative and more or less entertaining as determined by the character of the topic and the personality of the author. In the case of such books a poor collection may be worse than none, for these books may dictate and propagate an erroneous and otherwise vicious body of doctrine developed by the author, as no strictly reference work or tale of fiction is likely to do.

There is not the same need in the school library for a collection of books on mathematics that there is for reference books or volumes of lighter entertainment in other fields. No matter how popular mathematics may prove as a course of study when compared among other courses, few students will spend their spare time in wider reading in the subject-matter whatever may be the means of attraction employed. It is not a subject in which the student will seek material for a theme, essay, debate, or declamation. In a truer sense than in most subjects, the text used in the classroom is likely to be regarded by the scholar as self-contained and complete. For the child that regards voluntary reading as too tiresome because too often meaningless to be worth the effort, and such pupils are likely to form the majority of the class, it is useless to offer temptations of a mathematico-literary sort. There will seldom be a dispute that requires a mathematical encyclopedia for its settlement. And yet, despite all these obvious facts, for the practical preparation of the teacher, for the encouragement and development of the brightest pupils, for the clearing up of many minor points in which the text used is likely to be ambiguous, a mathematical library plays a very important, if not very obvious, rôle in the adequate management of the good school.

When the question arises as to the particular books that the library of a given school should possess, no very definite answer is easy to make. A short discussion as to several distinct types of books all of which make their appeal may not be out of place. Six special classes will be mentioned and others equally important may occur to the reader. These classes are not wholly independent, but will not be found to overlap extensively.

1. *Parallel Texts.* By this is meant other texts which cover the same general ground as the text used in the classroom. Some of the difficulties of a subject are due to the idiosyncrasies or to the carelessness of the author—to his use of peculiar or ambiguous phrases, his narrow choice of illustrations, his presumption that the reader is already familiar with some notion or notation used but not explained in the given text. In all of these circumstances, the language and viewpoint of an independent author are of the greatest service in clearing away these extraneous difficulties. As a minor but significant advantage in having access to a parallel text one early realizes the usefulness of having at hand a fresh supply of appropriate exercises not directly available to the pupil, when one faces the familiar task of making out an examination. Such parallel texts are easily available and are advertised on every hand. There is no need of mentioning particular names and titles, and there is often little to choose between competing texts. It is merely a case of two heads being better than one, so that even a very poor text used judiciously in comparison may bring out the full value of a much better book.

2. *Thorough Treatises upon the Same Ground.* A text adapted for high schools can and perhaps should leave many loose ends. There will be many topics touched upon or only hinted at that the pupil need not be required to investigate further. The teacher, however, should have a certain familiarity with the background of the subject that cannot be acquired by mere study of the accepted text. This background is naturally available in courses offered at college or in the university, but this does not mean that every teacher has the opportunity to acquire even the

needed preparation. Even if the appropriate course was taken, it may well happen that the application was not obvious at the time or that the subject-matter has been largely forgotten before its utility is realized. The policy of depending upon old notes to straighten out present difficulties is in any case not to be particularly encouraged. The wise and frequent use of a thorough standard text is one of the best methods of enriching the teacher's mind on matters directly pertaining to classroom studies. Furthermore the brightest pupils may ask leading questions which would unduly consume classroom time to discuss as they deserve, but which should be answered in a definite but enlightening manner. One of the best methods of dealing with such serious questions is to encourage as far as possible the inquiring mind not only by direct verbal response but by opening up still wider fields where the pupil may explore at will through the reference to a good text where this and many other related questions are treated in a logical and informative manner.

3. *Treatises on Topics Not in the School Curriculum.* It is impossible absolutely to separate the sciences and their subdivisions into distinct and independent topics. Yet in a general way one can assert even in a single science like mathematics that a given subject is or is not in the school curriculum. Mathematics enjoys a unity and at the same time a diversity, that is inadequately reflected in its elementary branches. To understand properly either geometry or algebra, one must seek to understand the bearings of each upon other mathematical topics. A too narrow field, however sharp the focussing or critical the examination, ruins the mathematical vision. For recreation of a sober sort and to attain to a scholarly breadth of view, few suggestions are more obvious than that one should browse around in neighboring fields where the pasturing is not less rich for being less frequently trodden down.

4. *Histories of Mathematics.* Probably many subjects that are the outcome of generations of mental labor, can only be adequately understood in the light of their historical setting. The chronological growth of a science is

likely to show some correlation to its logical structure, psychological graduation, and commercial applicability. That this is true of mathematics is obvious to all who have given this matter a thought. One should not, however, conclude that this obvious parallelism has any deep significance or has any mystical validity as a guide to teaching when one examines the subject in its finer details. A conscious emphasis upon historical order is perhaps doing today more harm than good in mathematical pedagogy. The best versed mathematical teachers of today, while familiar with the main historical features of their science, do not let this influence them in the order which they follow in presenting a topic. Psychological, logical, esthetic and other such non-temporal standards are their sole criterion. Though the history of mathematics has little if any claim to be regarded itself as mathematics, there is no doubt that a healthy human being with an alert mind has at times at least some curiosity and interest in learning how the subject he is examining came to have its present shape. Mathematics, unlike the younger sciences, has a history reaching back to the first traces of civilization. It should not be surprising that mathematical notions have been current since before the construction of the Egyptian pyramids, or the observations of the Babylonian astrologers, and that the political and cultural milestones in historical human progress should be reflected in this science no less than in art. Each of us as an heir of the past, guardian of the present and transmitter to the future has the privilege of learning how the cultural life of the present came to be.

5. *Books of Mathematical Applications.* The curious as well as the sullen student is sure to ask as to the usefulness of mathematics. No mere rhetorical reply will suffice. The question is always asked in a practical sense and although an idealistic answer may prove eventually the most satisfactory, it does not suffice for the instant. Three types of books which reply to this question come to mind. First, those which show the place of mathematical thought in the philosophical problems of civilization. These sometimes amuse the professional mathematician and sometimes fail

to impress the non-mathematician, but they surely have their place for the intelligent inquiring layman. Second, those works on astronomy, physics, mechanics, electricity, and other special physical sciences, and to a lesser degree other non-physical sciences, which bring to play on their problems whatever mathematical machinery may seem best adapted to the problem in hand, and which therefore assume a fair or even intensive mathematical preparation on the part of the reader. For the unfortunate experimenter who cannot follow the mathematics involved, such books are usually a despair, although they sometimes lead to serious misconceptions. The conclusions and even the reasoning may be capable of much more familiar, if less elegant expression, so that not a few non-mathematicians who have at last grasped a simple mathematical notion, but who have never learned to think in mathematical manner, proceed to rush into print as though with a wonderful discovery, and merely explain an elementary mathematical concept while refusing to avail themselves of the machinery that through countless ages man has been perfecting for this very purpose, and in terms of which the whole problem may reduce to a simple exercise. The work of exposition may mean something to the author but it may fail to mean anything to anyone else. Thus these technical works and their "non-mathematical" travesties, both reveal the usefulness of mathematics as applied in research. Third, books of an elementary nature that observe a logical development of mathematical notions, but feature numerous applications of these abstract principles to some particular science. Such a text is practical to the student of the particular subject whose mathematical intuition or preparation is so poor that a verbal problem does not suggest the mathematical formulation as solution. To the student who knows nothing of the science in which applications of mathematics are being made, the whole book will be meaningless except in so far as he is likely to admit that applications must surely exist in this unfamiliar field. To the able mathematician the book may be a bore or even an insult, since the applications may be regarded as extraneous to the real subject-matter of interest, and the explanation as to how to formulate the mathematical expressions required sometimes implies that the reader is almost submoronic in intelligence. However, these books have their place and fill a need felt by the student in his less docile moments.

6. *Books on Mathematical Recreations.* These contain more or less mathematics and although their appeal may not be a very serious one, it makes up in liveliness what it may lack in depth. The mere working over what others have done in satisfactory form is no test of mathematical ability, and were it not for the creative development of the science it would be unworthy of the lifetime devotion of the great minds identified through the centuries with its progress. The appeal of the unknown, the lure of the puzzle, characterizes those problems on which mathematical genius of today is devoting its efforts. In a more or less childish form, these same features are found in the mathematical recreations, some of which have a long and celebrated history. Such books may actually awaken mathematical interest that would otherwise remain dormant, but in any case, when not overstressed they serve a useful purpose in affording innocent and somewhat instructive amusement to many pupils.

The following is a list of forty-two books suitable for a school library, and exclusive of parallel texts discussed above. The list is not exhaustive and does not even pretend to present the cream of the various fields touched upon. For obvious reasons the numerous excellent books in French, German, or Italian, that are not difficult to obtain and that frequently conspicuously surpass the best in the same line that are to be found in our own language, are not here listed. Possibly one or two here mentioned may be out of print, and many of them are printed in Great Britain, but they are probably all available with at most a short delay, through any bookseller. The books mentioned are selected from those found here in the University Library shelves, and are not based upon a careful survey of the present publishers' catalogues.

The list is arranged alphabetically by authors, since the character of any given text and the occasion for its inclusion here will be obvious from the remarks made above.

1. Abbott, E. A., *Flatland.* Little, Brown & Co.
2. Ball, W. W. R., *A Short Account of the History of Mathematics.* The Macmillan Co.
3. Ball, W. W. R., *Mathematical Recreations and Essays.* The Macmillan Co.
4. Bonola, R., *Non-Euclidean Geometry.* Open Court.
5. Breslich, E. R., *Correlated Mathematics for Junior Colleges.* University of Chicago Press.
6. Cajori, F., *A History of Elementary Mathematics with Hints on Teaching.* The Macmillan Co.
7. Carmichael, R. D., *Theory of Numbers.* Wiley.
8. Carson, G. E. St. L.,, *Essays on Mathematical Education.* Ginn & Co.
9. Casey, J., *Sequel to Euclid,* Longmans, Green & Co.
10. Chrystal, G., *Algebra* (2 vols.). A. and C. Black (Edinburgh).
11. Clay, C. M., *Examples in Algebra* (8000 examples). Ginn & Co.
12. Dedekind, J. W. R., *Essays in the Theory of Number.* Open Court.
13. Dudeney, H. E., *Amusements in Mathematics.* Nelson (London).
14. Fine, H. B., *Number System of Algebra.* D. C. Heath & Co.
16. Gallatly, W., *Modern Geometry of the Triangle.* Hodgson.
17. Hobson, E. W., *Plane Trigonometry.* Cambridge University Press.
18. Hobson, E. W., *Squaring the Circle.* Cambridge University Press.
19. Hudson, H. P., *Ruler and Compasses.* Longmans, Green & Co.
20. Huntingdon, E. V., *The Continuum.* Harvard University Press.
21. Hyde, E. W., *Grassmann's Space Analysis.* Wiley.
22. Keyser, C. J., *The Human Worth of Rigorous Thinking.* Columbia University Press.
23. Klein, F., *Famous Problems of Elementary Geometry.* Ginn & Co.
24. Koch, E. H., *The Mathematics of Applied Electricity.* Wiley.

25. Lachlan, R., *An Elementary Treatise on Modern Pure Geometry.* The Macmillan Co.

26. Lagrange, M. J., *Elementary Mathematics.* Open Court.

27. Licks, H. E.,, *Recreations in Mathematics.* Van Nostrand.

28. Manning, H. P., *Geometry of Four Dimensions.* The Macmillan Co.

29. Manning, H. P., *Non-Euclidean Geometry.* Ginn & Co.

30. Mellor, J. W., *Higher Mathematics for Students of Physics and Chemistry.* Longmans, Green & Co.

31. Phin, J., *The Seven Follies of Science.* Van Nostrand.

32. Poincaré, H., *The Foundations of Science.* The Science Press.

33. Putnam, G. E., *The Mathematical Theory of Finance.* Wiley.

34. Row, T. S., *Geometric Exercises in Paper Folding.* Open Court.

35. Schubert, H., *Mathematical Essays and Recreations.* Open Court.

36. Smith, D. E., *History of Modern Mathematics.* Wiley.

37. Smith, D. E., *The Teaching of Geometry.* Ginn & Co.

38. Thorndyke, E. L., *The Psychology of Algebra.* The Macmillan Co.

39. Whitehead, A. N., *An Introduction to Mathematics.* Cambridge University Press.

40. Woodward, R. S., *Probability and the Theory of Errors.* Wiley.

41. Young, J. W., *Fundamental Concepts.* The Macmillan Co.

42. Young, J. W. A., *Monographs on Topics of Modern Mathematics.* Longmans, Green & Co.

BY ALBERT A. BENNETT

THE GREATEST VARIATE IN CERTAIN DISTRIBUTIONS*

BY EDWARD L. DODD

In statistical studies we are often interested, not only in the average size or measure of the objects or phenomena under consideration, but in the extreme sizes, the very large or the very small size. Insurance companies commonly reject for insurance a man who is an extreme underweight or an extreme overweight. Thus, the heaviest man among one thousand insured men taken at random would probably weigh less than the heaviest among one thousand uninsured men taken at random.

By a variate we mean any one of the values which a variable may take on. Thus we may let w stand for weight; and w_1, w_2, w_3, etc., for the weights of individuals in the class considered. Then w is the variable, and w_1, w_2, w_3, etc., are the variates. We wish to consider here to what extent and in what manner mathematics may be used to obtain the value of the greatest variate.

We first suppose that the distribution of sizes or measures follows some law that can be expressed mathematically. Let us illustrate this by a simple problem in coin-tossing, and suppose that a coin is tossed four times. Then, distinguishing the different trials in which "heads" can appear, the possibilities are as follows:

Possible number of heads	0	1	2	3	4
Number of ways of getting the specified number of heads	1	4	6	4	1
Probability of getting that number	1/16	4/16	6/16	4/16	1/16

*Read before the mathematics faculty and invited friends, December 6, 1923.

Those familiar with the theory of probability will recognize at once that the last row of numbers are the terms in the expansion of $(\frac{1}{2}+\frac{1}{2})^4$. Here $\frac{1}{2}$ is the probability of getting "heads" in a single trial—also of not getting "heads," and 4 is the number of trials. We may represent this graphically by marking off 0, 1, 2, 3, and 4 as abscissas, and erecting ordinates proportional to the "frequencies" 1, 4, 6, 4, and 1, respectively. These same ordinates we may think of as probabilities, "relative frequencies" if the vertical scale is increased sixteenfold. We obtain now a "frequency curve" by drawing a smooth curve through the extremities of the ordinates. Even when n, the number of trials, is as small as 4 we get a bell shaped curve, a shape commonly found in biological distributions and to a considerable extent in sociological distributions. But if the coin is tossed a large number of times, and the horizontal and vertical scales are properly adjusted, the frequency curve approaches a limiting form, given by the Gaussian probability function

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

The origin now corresponds to $n/2$; x is a "deviation" from $n/2$, the number of "heads expected" in n trials; and σ is the "standard deviation."

$$\sigma = \frac{\sqrt{n}}{2} \tag{2}$$

The probability that in n trials the number of heads will not exceed $\frac{n}{2} + x$ is given with good approximation, by

$$\int_{-\infty}^{x} y\, dx. \tag{3}$$

The foregoing curve, commonly called the *probability curve,* was found to describe frequency distributions in the physical sciences with such accuracy that for a long time little attention was paid to any other curve to describe frequency. But, with the growth of the biological and sociological sciences, a demand for other curves arose. Now, the Gaussian probability function in .1) satisfies the differential equation

$$\frac{1}{y}\frac{dy}{dx}=\frac{-x}{\sigma^2}. \tag{4}$$

Karl Pearson generalized this, writing

$$\frac{1}{y}\frac{dy}{dx}=\frac{-x+k}{a+bx+cx^2}, \tag{5}$$

and thus obtained a variety of curves or "types" which have proven very serviceable in fitting frequency distributions. Several other curves have been used by statisticians, some of which are modifications of the common probability curve. To describe frequency of telephone calls, the Poisson Exponential is generally used; viz.,

$$y=\frac{e^{-\lambda}\lambda^x}{x!}. \tag{6}$$

The Charlier B— series is obtained from this by differencing. For the duration of human life, the Makeham Function is commonly accepted; viz.,

$$y=ks^xg^{c^x}, \tag{7}$$

where x is the age of an individual, and k, s, g, and c are constants.

The foregoing frequency functions and, indeed, functions much more general than these, I have studied with reference to the problem of determining the greatest in a set of n variates. The results are to be found in the Transactions of the *American Mathematical Society* for October, 1923. I wish here to sketch the method of approaching the problem.

In general, the size of the greatest variate will depend upon the "population" n in which it appears. The oldest man in Austin is *probably* not as old as the oldest man in Chicago. We say "*probably*" because the whole problem is a phase of probability. However, there are probabilities which converge toward certainty. The following is an illustration: A coin, with ½ as probability for "heads," is tossed n times.

n Number of tosses of a coin	P Probability that "heads" will appear in at least 48 per cent of the tosses and in not more than 52 per cent.
100	.31
1,000	.79
10,000	.9999
40,000	.999,999,999,9

Here

$$\lim_{n \to \infty} P = 1. \tag{8}$$

This is not merely a plausible conclusion from the large number of 9's that appear above, but a rigorous conclusion from the hypothesis that a primary probability of ½ is involved n times in a process called repetition.

The coin is really only a superfluous physical ornament to the problem. Whether a coin exists, with ½ as probability for "heads," has no significance for mathematics. In geometry we do not concern ourselves with the physical existence of mathematical planes, cones or spheres. In fact, mathematics is an abstract science; and our cones and coins exist only in our own imaginations.

Thus, the problem of the greatest variate for large classes of frequency functions can be solved *rigorously* by the use of calculus, various inequalities, and the theory of limits. We suppose given two small positive numbers, η and ϵ, as small as we please. Inasmuch as certainty is indicated by the number *1*, it follows that $1-\eta$ will indicate a probability little short of certainty if η is taken very small. The ϵ here will represent the maximum error permitted for an approximation. Very occasionally it is a maximum *abso-*

lute error, usually, it is a maximum *relative error;* sometimes, it is a maximum relative error for an exponent in the approximation, instead of for the approximation as a whole. In the following theorem, it is maximum relative error. This theorem covers as a very special case the usual probability function.

Theorem

Suppose that, for each of n variates, the probability that it will be less than x is

$$\int_{-\infty}^{x} \phi(x)\,dx,$$

where $\phi(x) \geqq o$, and satisfies the requirement

$$\int_{-\infty}^{\infty} \phi(x)\,dx = 1. \tag{9}$$

Suppose, moreover, that $\phi(x)$ has the form

$$\theta(x) = g^{x^{\alpha}} . \psi(x), \tag{10}$$

with $o < g < 1$, $\alpha > o$ and $\psi(x)$ such a function that for all values of x greater than some positive constant,

$$x^{-\beta} < \psi(x) < x^{\beta}, \text{ with } \beta > o. \tag{11}$$

Then, if two positive numbers, η and ϵ, are chosen, small at pleasure, it is possible to find n' so that if $n > n'$ there is a probability greater than $1 - \eta$ that the greatest variate will be

$$(-\log_g n)^{\frac{1}{\alpha}} . (1+\epsilon'), \text{ where } |\epsilon'| < \epsilon. \tag{12}$$

This theorem, applied to the Gaussian probability function (1) may be stated briefly, as follows:

If each variate in a set of n variates is subject to the Gaussian probability law

$$y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

then with increasing n, it becomes asymptotically certain that the greatest variate G will be

$$G = \sigma\sqrt{2 \log_e n}, \tag{13}$$

with a relative error small at pleasure.
This

$$\sigma\sqrt{2 \log_e n}$$

may be properly called an asymptotic value of the greatest variate under the Gaussian probability law.

Another theorem gives the following result: Under the Makeham Life Function (7) the age of the oldest of n individuals is given asymptotically—ie., for large n—by

$$G = \log_c (-\log_g n). \tag{14}$$

Using c and g as computed for the American Experience and McClintock Annuitant Tables, for populations varying from one thousand to one billion, I found that this asymptotic formula gave ages from 95 to 113 years. However, a population of one thousand is rather small to which to attempt to apply an asymptotic formula.

Using the normal probability function, Bortkiewicz* determined mean and modal values for the interval of variation, *i.e.*, the difference between the greatest and the least of the n variates. The *mean* value here refers to the ordinary average or arithmetic mean; the *modal value* is the most probable value. Bortkiewicz also showed that his formulas gave very good approximations to certain data obtained from roulette, and also from measurements of skulls. Statisticians also use the *median value, m.* Here m is the median value of the greatest variate G, if it is equally likely

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*"Variationsbreite und mittlerer Fehler" — Sitzugsberichten der Berliner Mathematischen Gesellschaft XXI Jahrgang, Sitzung am 26. Oktober, 1921.

that G will lie above m or below m. This median value can be found by determining m so that

$$\int_{m}^{\infty} y\,dx = 1 - 2^{-\frac{1}{n}} \tag{15}$$

granted that, for each variate,

$$\int_{x}^{\infty} y dx \tag{16}$$

is the probability that it will be greater than x. Using this formula, I computed median values for the interval of variation and found that there lay between the mean and modal values calculated by Bortkiewicz. The median should, indeed, lie between the mean and the mode.

The asymptotic value of the interval of variation in the case of any symmetric law like the Gaussian law is double the asymptotic value of the greatest variate. For the Gaussian law, the asymptotic value of the interval of variation is 23 per cent greater than the median value when $n=100$; it is only 10 per cent greater when $n=100{,}000$.

Asymptotic values give rough checks upon mean, median, and modal values, the latter three values being, in general, more difficult to compute.

But the chief characteristic of an asymptotic value is that there is asymptotic certainty that the asymptotic value will be approximated, with a relative error small at pleasure.

THE NEGLECTED MOTHER OF SCIENCE

By McN. Simpson, Jr., Randolph-Macon College, Ashland, Virginia

Priority among the sciences is rather generally conceded to astronomy. It is not strange that this is so. Primitive people were out door people. Nature in the large rather than in its minutiæ was the object of observation. The five senses were the sole laboratory equipment. Inner structures, whether of inanimate or animate things, were mysteries for a long while unexplored.

But the first man had a sky to study not differing much from the sky above our heads and it was far less obstructed by artificial roofs than ours. There was grandeur about it and mystery about it but there was evidently order about it. One could count on rising and setting suns, and the grand procession of the constellations; one might be less certain about rain and wind and heat and cold. Far before any date that we can set came the inspiration of genius that sought to reduce the order of the sky to law, to seek causes for effects.

And so science began. Of course occultism came too. That has been the apparently inevitable fate of all science, ancient and modern. And so astrology was born and perhaps there is too much astrology and too little astronomy in some of the fragments that have come to us. But chemistry must bear the reproach of some of alchemy's vagaries, and even modern medicine must protect itself against fads and panaceas.

Why the modern neglect of astronomy? Why the staggering ignorance of the masses, educated as well as uneducated, of the simple facts of this most ancient science? Why has a curriculum, which in its ancestral form numbered astronomy among the four essential subjects, crowded it out of place with new but more instructive or more inspiring successors? Why is the mother of science belittled?

shadow, of rainbow and aurora, of comet and meteor, of tenuous nebula and majestic star clusters. Still, inspiring guide, she stimulates our eagerness to break the bonds of ignorance and let the truth free—truth just for its own sake.

The literal minded may object that infinitudes and infinitesimals are alike materially unthinkable, but the astronomer will have little quarrel with the poetic fancy that sees our universe as "boundless inward in the atom, boundless outward in the whole."

TEACHING THE SIMPLE EQUATION IN ALGEBRA

Too many teachers of algebra explain their failure in successfully putting across the written simple equations by declaring that the equations are too difficult for eighth and ninth graders, and should be eliminated from serious consideration by teacher and pupils. To disregard the importance of these beginning equations is to later disregard fractional equations, equations with two unknowns, and possibly indeterminate equations. Undoubtedly, if algebra has a real value in the student's training, that value is incorporated in the intelligent analysis and consequent solution of the various written equations. To omit them is to cut the heart out of this study. And it is evident that the pupil will not master, nor be interested in the written equations, unless he shall have had proper training in the simple equations, and have felt that pleasure of achievement, of surmounting a real task.

As an introduction to the simple equations a teacher should emphasize two ideas. First, that simple equations contain one thing and only one, which if known, would make the solution of the problem obvious by arithmetical procedure, but since there is some one thing not known, we merely substitute a letter or symbol for the unknown until it can be ascertained. Second, that an equation consists of two equal expressions, one of which involves the unknown. A single explanation will not suffice. It is necessary to bring these points out day after day until the pupils have thoroughly understood them. "Why certainly," some will say, but the fact is indisputable that many teachers fail to impress these basic principles upon the young minds in the class.

Once the pupil understands what he wants to look for in the problem you have his interest aroused, without which no results can be obtained. There is a natural coördination between algebra and arithmetic, and the teacher should avail himself on every occasion of the opportunity to coördinate the two. Before trying to solve problems concerning distance, it is very necessary that the pupil understand rate, time and distance, and the relation of each to the other two, with numerous examples from the field of arithmetic. In problems relating to age every statement relative to future or past time affects both parties of the problem. In the problems which deal with cisterns have the class solve many similar arithmetical problems concerning cistern problems similar to these before introducing the simple equation of algebra. Then when dealing with problems involving money teach them that value on one side of the equation and number of coins on the other do not harmonize; that there must be "number of coins" represented on both sides of the equation, or value on both sides. And so on with other well known varieties of problems in the text.

Again, there is so much to gain from analysis. Solving a problem should be the last step. The solution should never begin until the final disclosure is made in the analysis. And right here is an opportunity to let rivalry play its part. Have the students quietly read the problem and allow so much time for hands to be raised by those having found the unknown in the problem; which if known "you would know this, and if this is known, we would also know this," and so on. This will develop interest among the individuals, and will lead to constructive thinking.

Although a few suggestions have been offered, no instruction has been intended. Rather have my thoughts been on the necessity of teaching the written simple equations, and to suggest to teachers relegating the equations to the background that a more logical presentation might help. Individuality of teacher plus enthusiasm will achieve more beneficial results than all the pet rules any one teacher may lay down. The main thing is to get on the job.

APPROXIMATE CONSTRUCTIONS OF REGULAR INSCRIBED POLYGONS

It is shown in books on Theory of Equations that it is impossible to construct (with theoretical accuracy) by the use of the ruler and compasses alone regular inscribed polygons of seven sides and nine sides.

It then becomes an interesting problem to construct approximately these polygons with as small an error as possible and at the same time employing constructions which are fairly simple.

In what follows the constructions are quite simple.

Regular polygon of 7 sides.

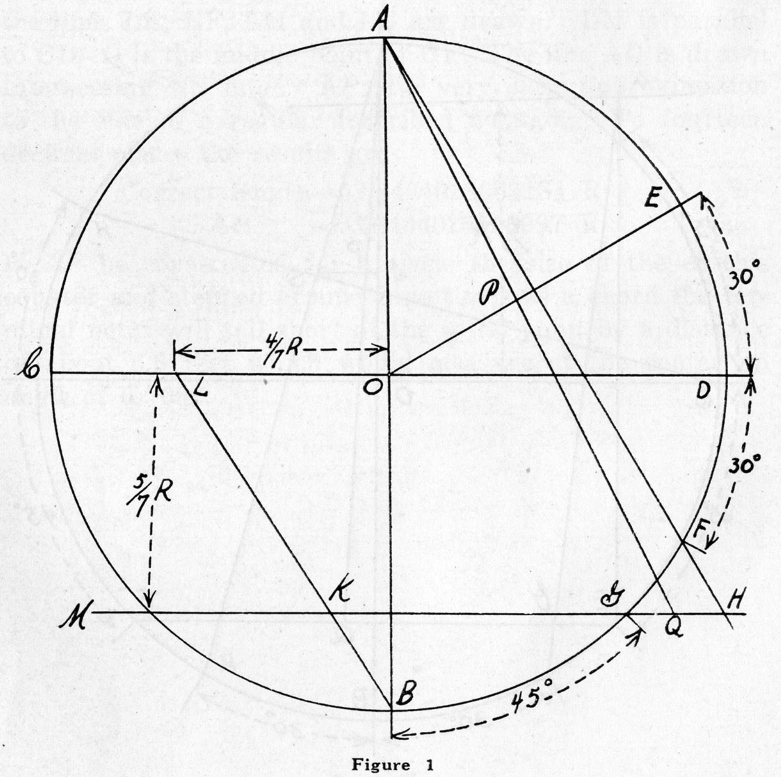

Figure 1

Figure 1 gives an approximate construction of a regular inscribed polygon of seven sides. A circle whose center is at 0 and radius R is drawn. AB and CD are two perpendicular diameters. The point L is determined as indicated and the line LB is drawn. The line MK is drawn parallel to the diameter CD. From the intersection K, of LB and MK the line KH is drawn, intersecting AH in H. Q is the middle point of GH. AQ is drawn intersecting OE in P.

AP is a very close approximation to the side of a regular inscribed heptagon. It must be remembered that MK and KH do not form the same straight line. To fourteen decimal places the correct length of the side and the length obtained in the construction are given.

Correct length$=0.86776747823512$ R

AP $=0.86776747796535$ R

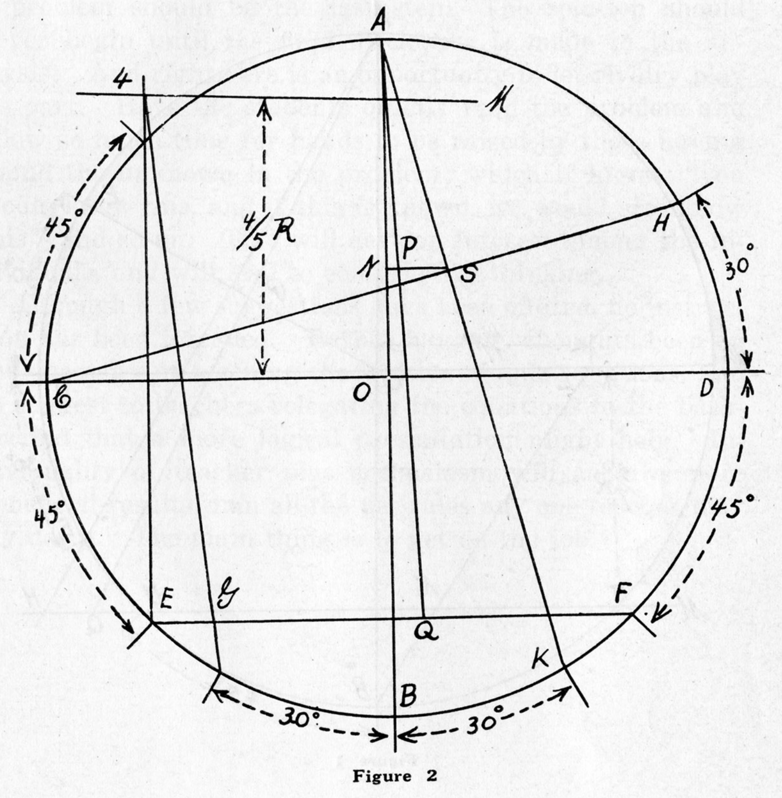

Figure 2

For the purpose of better showing the closeness of the approximation imagine AP constructed for a circle the size of the earth's equator and laid off as a chord seven times. The terminal point would fall short of the initial point by 0.53 inch or an arch which measures at the center an angle of $0.''000\ 43$. That is to say, the polygon would fail to close by 0.53 inch.

Regular polygon of 9 sides.

Figure 2 shows the approximate construction of a regular inscribed nonagon.

A circle whose center is 0, and radius R, is drawn. AB and CD are two perpendicular diameters. The figure explains itself. First the lines CH and AK are drawn intersecting in S. The line SN is drawn parallel to CD. Now the lines LE, EF, LM and LG are drawn. LM is parallel to CD. Q is the middle point of GF. The line AQ is drawn intersecting NS in *P*. AP is a very close approximation to the side of a regular inscribed nonagon. To fourteen decimal places the results are:

$$\text{Correct length} = 0.68404028665134\ R$$
$$AP = 0.68404025506097\ R$$

If AP be constructed for a circle the size of the earth's equator and stepped around nine times as a chord the terminal point will fall short of the intial point by a distance of about 6.3 feet which would measure at the center an angle of $0.''062$.

LADYLIKE GEOMETRY

(NELLIE PARKER JONES, in the *Chicago Record-Herald*)

1. A *straight line* is the shortest distance between two millinery openings.

2. A *straight line determined* by two bargain tables is considered as prolonged both ways until the store closes.

3. A *broken line* is a series of successive straight lines described by a woman alighting from a street car.

4. A *mixed line* is a line composing the reception committee of a club's presidential candidate.

5. A *plain figure* is one all points of which have been neglected by the dressmaker.

6. Figures of the same shape do not always have the same style.

7. Figures of the same size never consider themselves equivalent.

8. Women equal to the same thing are not always equal to each other.

SERVICE MATHEMATICS

BY MARY E. DECHERD

In connection with the recent meeting of the Western Section of the American Mathematical Society in Columbia, Missouri, December 1, 1923, the Missouri section of the Mathematical Association of America was also in session and presented a most interesting program. One of the speakers was Mrs. Theodosia Tucker Callaway of Stephens Junior College, Columbia, Missouri. Her subject was extremely practical and the conclusion at which she arrived added greatly to the importance of her paper.

For some months Mrs. Callaway has been investigating the character of mathematics needed for the home economics department of the college in which she teaches. Her method of investigation is the examination of the various texts used in the home economics department, listing all mathematical terms and processes and determining the frequency of their occurrence. The textbooks examined were those used in chemistry, foods, and clothing.

In the first table Mrs. Callaway found that concepts employed were 255 in number. For the most part while the terms are strictly speaking mathematical, they are common in ordinary conversation. For example, the terms add, center, curve, divide, equal, length, line, number, point, quantity, volume have the highest frequencies. Indeed there are only six words with which an eighth-grade student could not be expected to be familiar. A second table on the denominate numbers used in these subjects exhibits 100 terms and I fear that the student will not be as well acquainted with these as with the terms in the first table. Here are found both, French metric system and English denominate numbers as well as the more usual concepts such as, feet, cups, dollar, teaspoon, etc. When it is recalled that the "Brazilian schools save a year in arithmetic by employing only the metric system," cannot one be excused for a feeling of impatience that the United States still insists on using two systems?

However, even here the situation is not difficult for about one-fourth of the 100 terms are abbreviations of terms counted in the hundred, e.g., gallons, gal.; dozen, doz.

A third table is concerned with the common fractions used in the texts examined. As one would expect the most frequently used fractions are ½ used 815 times, ⅓ used 106 times, ¼ used 474 times, ¾ used 169 times, and ⅛ used 149 times.

The fourth table devoted to decimal shows that in only a very few instances does the subject call for the use of numbers with more than four figures.

Hence the author arrives at the conclusion that what the home economics students need is not college mathematics, not algebra, nor geometry, nor analytics, nor trigonometry but *mirabile dictu!* only the arithmetic that is given in the course of study in our elementary schools. She quotes a student in a home economics class who announced that she knew how to add fractions, but that she did not know when to add them. Finally, then, she said, that what the home economics departments needed was not then that pupils should have more mathematics nor yet that the subject-matter given them should be different, but that the mathematics now given in the elementary schools should be taught more efficiently. What do we teachers of mathematics say in reply to the conclusion reached by one of our own number?

SOME SUGGESTED PROOFS IN GEOMETRY

By J. E. Red

In the proofs of Propositions XI and XII, Book IV of *Wentworth and Smith's Plane Geometry,* a somewhat arbitrary method has been used by the authors. While such a method is at all times admissible and often necessary, and does not violate any fundamental mathematical principles, it is, however, not at all necessary in the proofs of these propositions and is foreign to the method underlying the solution of same.

If it can be shown that each step in a proof is based upon some definite geometrical relationship, and that the substitutions which are necessary to make the final equation conform to all the conditions of the theorem are made from the figure, the student will have a firmer grasp of the subject in that he has made adequate application of established facts in the form of theorems, corollaries, *et cetera.*

In Proposition XI, the proof is begun with the equation $A'=b'+c$ (See text). These equations are evident from the figures. But in the next step in the proofs an algebraic transformation is made by squaring these equations. Then a purely arbitrary operation is made in the third step by adding h^2 to both sides of the resulting equations. It is my opinion that it would be better to rely upon the Pythagorean theorem for the basic data of the proofs.

The suggested proofs are given below.

Proposition XI

In any triangle the square on the side opposite an acute angle is equivalent to the sum of the squares on the other two sides *diminished* by twice the product of one of those sides by the projection of the other upon that side.

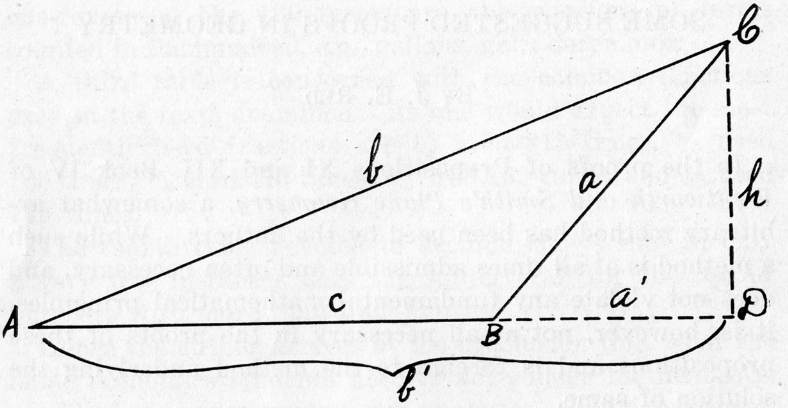

Figure 3

Given the triangle ABC, A being an acute angle, and a' and b' being the projections of a and b respectively upon C.

To prove that $a^2 = b^2 + c^2 - 2b'c$.

Proof.—Draw h from $C \perp$ to AB produced to D.

$a^2 = a' + h^2$ (Sec. 337, text) (1)

But $a' = b' - c$ (2)

$\therefore \quad a^2 = (b' - c)^2 + h^2 = b'^2 + c^2 + {}^2 - 2b'c$ (3)

Again, $b^2 = b'^2 + h^2$ (Sec. 337, text) (4)

Substituting in (3) b^2 for its equal

b'^2 h^2, we have

$a^2 = b^2 + c^2 - 2b'c$.

Proposition XII

In any obtuse triangle the square on the side opposite the obtuse angle is equivalent to the sum of the squares on the other two sides *increased* by twice the product of one of those sides by the projection of the other upon that side.

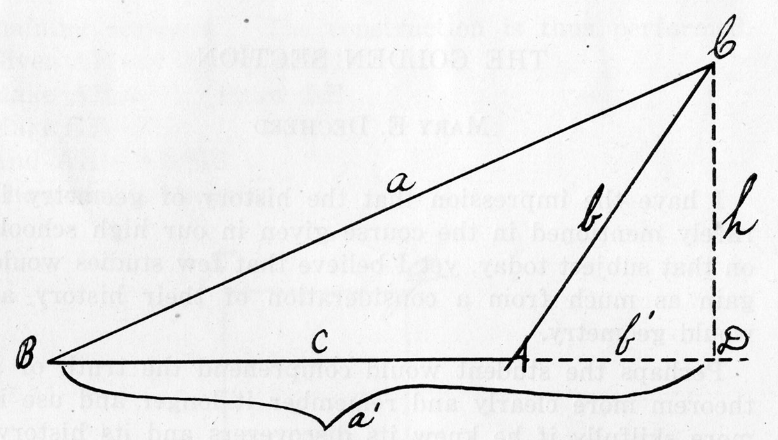

Figure 4

Given the obtuse triangle ABC, A being the obtuse angle, and a' and b' being the projections of a and b respectively upon c.

To prove that $a^2 = b^2 + c^2 + 2b'c$

Proof.—Draw h from $C \perp$ to AB produced to D.

$$A^2 = a'^2 + h^2 \text{ (Sec. 337, text)} \qquad (1)$$

But $a' = b' + c$ (2)

$$\therefore \quad a^2 = (b'+c)^2 + h^2 = b'^2 + c^2 + h^2 + 2b'c \qquad (3)$$

Again, $b^2 = b'^2 + h^2$ (Sec. 337, text) (4)

Substituting in (3) b^2 for its equal

$$A^2 = b^2 + c^2 + 2bc.$$

Q. E. D.

THE GOLDEN SECTION

MARY E. DECHERD

I have the impression that the history of geometry is rarely mentioned in the course given in our high schools on that subject today, yet I believe that few studies would gain as much from a consideration of their history as would geometry.

Perhaps the student would comprehend the truth of a theorem more clearly and remember it longer and use it more skilfully if he knew its discoverers and its history.

May I say a few words about Proposition XII, Book V, page 244 of *Wentworth* and the associated Proposition XXVIII, Book III, page 185?

Some authorities think that this latter problem was first proved by Eudoxus, a friend of Plato, 370 B.C. It was he who discovered that the solar year was six hours longer than the previously determined 365 days, and he is said to have made a sun dial. Eudoxus called his division of a line in "extreme and mean ratio" the golden section. Other writers, however, contend that Plato himself first divided the line in extreme and mean ratio; and also Hippocrates in the fifth century B.C. is said to have solved this problem. At least, of this much we may feel reasonably certain: the Pythagoreans were familiar with the solution of this and similar problems. It was Euclid, the "father of geometry" who arranged the "discoveries of Eudoxus" and of the Pythagoreans and made various applications of them.

Proposition VI, 30 in "The Thirteen Books of Euclid's Elements" is "to cut a given finite straight line in extreme and mean ratio." It happens that the method actually employed here depends on a more general theorem concerning parallelograms. However, the same construction follows immediately from Euclid's Proposition II, 11, "to cut a given straight line so that the rectangle contained by the whole and one of the segments is equal to the square on the re-

maining segment." The construction is thus performed.
Given AB and the Sq. ABCD
Make AE=CE. Draw BE
Make EF=EB
And AH^2=AB.HB
The proof is obvious.

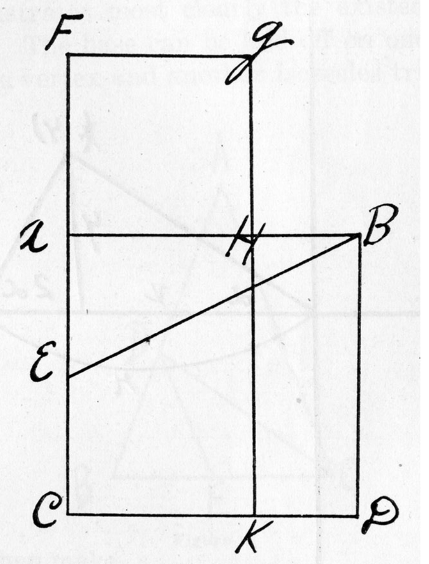

Figure 1

Wentworth's method of constructing a regular decagon is of course to divide the radius in extreme and mean ratio, the larger segment being the side of the regular decagon. Connecting alternate vertices of the regular decagon we obtain a regular pentagon.

Several other methods for this construction may be of interest.

Euclid, Proposition IV, 11 constructs the pentagon first, employing the isosceles triangle having one base angle double the other, this construction itself depending on Proposition II, 11 quoted above.

It is apparent that these geometric constructions are one with the solution of the algebraic equation $x^2—ax—a^2=0$.

Hence the side of a regular pentagon can be thus obtained as the intersection of two loci:

Take a circle of radius r. Now find the locus of a point which will be the vertex of a triangle having one base angle double the other as follows:

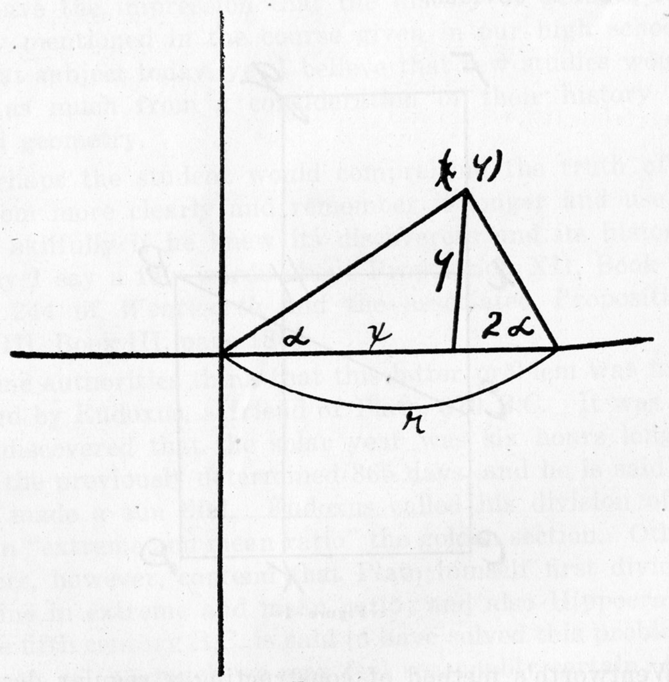

Figure 2

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\frac{y}{r - x} = \frac{2\frac{y}{x}}{1 - \frac{y^2}{x^2}},$$

and finally,

$$3x^2 - y^2 = 2rx.$$

The equation of the circle is $x^2+y^2=r^2$.
The point of intersection of these two loci will be one vertex of the decagon, a second one being the intersection of the circle and the x-axis.

It might be remarked in passing that the isosceles triangle which has its base angles double the angle at the vertex demonstrates most clearly the existence of incommensurables. The base can be laid off on one of the equal sides from the vertex and another isosceles triangle formed thus:

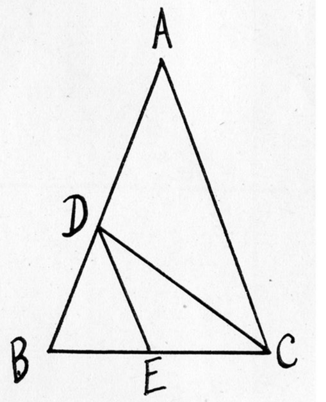

Figure 3

AD=BC, then make
EC=DB, etc.

As this process is continued a remainder always exists.

A simple method for determining the length of the side of a decagon is obtained thus:

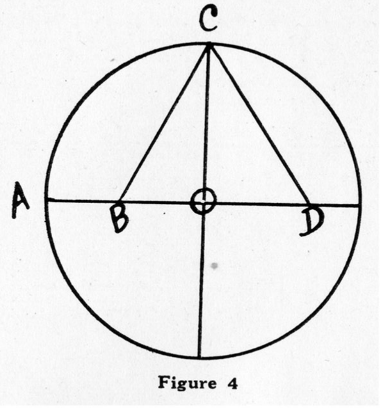

Figure 4

Within a circle draw two perpendicular diameters. Let AB=BO. Draw BC. Make BD=BC. OD is the side of the decagon and CD of the pentagon.

For $BC = \frac{r}{2}\sqrt{5} \therefore OD = \frac{r}{2}(\sqrt{5}-1)$ and $CD = \frac{r}{2}\sqrt{10-2\sqrt{5}}$.